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An Exploration of Fuzzy Sets and Their Fundamental Properties

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Abstract

In this paper we will study various types of fuzzy sets such as convex fuzzy sets, strongly convex fuzzy sets, strictly convex fuzzy sets, closed fuzzy sets. Also we will study some of the properties of above mentioned fuzzy sets.

Keywords: Fuzzy set, Convex Fuzzy Set, Strictly Convex Fuzzy Set, Closed fuzzy Set, Strongly Convex Fuzzy Set, α-cut, fuzzy hypograph**.**

Introduction

Fuzzy sets have many uses in both social and scientific disciplines, making them an area of research that is expanding. Professor Lotfi A. Zadeh's[1] 1965 paper, "Fuzzy Sets," introduced the idea of a fuzzy set. Later, this idea was used by several researchers to do additional research. Fuzzy set is an extension of Classical set. Xinmin Yang[2] has given the condition when a closed fuzzy set is a convex fuzzy set. Xinmin Yang[3] investigated the characteristics of convex fuzzy sets, strongly convex fuzzy sets, and strictly convex fuzzy sets. Yu-Ru Syau[4] defined fuzzy hypograph to show some of its properties, he provided conditions under which a fuzzy closed set is a convex fuzzy set, a strongly convex fuzzy set, or a strictly convex fuzzy set. Jozef Drewniak[5] shown properties of various notions such as core, support of fuzzy sets of convex and strongly convex fuzzy sets. The current research is on connecting different kinds of fuzzy sets and examining relations between them. Convexity is a key concept in mathematics with numerous applications in the sciences, industries, and the arts. Most things in our immediate environment are ambiguous in nature. To optimize the values from an application standpoint, it is vital to investigate the convexity of such phenomenon. The first attempt at applying the notion of convexity to fuzzy sets has been made by Zadeh. He has provided ideas about convexity. This paper focuses on study of various types of convexity and their relations with each other.

Preliminary

Throughout this paper B will denote Fuzzy set defined on R. J is closed interval 0 and 1. Here are some definitions that will be useful in this paper.

Definition

Let B be a fuzzy set defined on the set of real numbers together with its membership grades. A membership function is always present when a fuzzy set is thought of. It is denoted by $B : R \rightarrow$ J. $B(x_1)$ is a membership of x_1 in fuzzy set B.

Definition

$\alpha - cut$ of fuzzy set B is a collection of all elements from domain of fuzzy set B for which membership grade is greater than or equal to α. It is denoted by B_α.

Definition

A fuzzy set B on R is said to be convex fuzzy set if

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$B(\lambda x_1 + (1-\lambda)x_2) \geq min(B(x_1), B(x_2))$. $\forall$ x_1 *and* x_2 in R and $\lambda \in (0, 1]$.

Definition

A fuzzy set B on R is said to be strongly convex fuzzy set if

$B(\lambda x_1 + (1-\lambda)x_2) > min(B(x_1), B(x_2))$. $\forall$ x_1 *and* x_2 in R and $\lambda \in (0, 1)$ and $x_1 \neq x_2$.

Definition

A fuzzy set B on R is said to be Strictly convex fuzzy set if $B(\lambda x_1 + (1-\lambda)x_2) > min(B(x_1), B(x_2))$. $\forall$ x_1 *and* x_2 in R and $\lambda \in (0, 1)$ and $B(x_1) \neq B(x_2)$.

Definition

A fuzzy set B on R is said to be fuzzy closed set if for any $\beta \in J$, B_β is a closed set.

Definition

A fuzzy hypograph of a fuzzy set B defined on R denoted by f.hypo(B)

f.hypo(B) = $\{(y, t') : y \in R, t' \in (0, B(x))\}$.

Definition

A fuzzy set B on R is closed if and only if B is upper semi continuous on R.

Main Results

In this section we will put forward some Results on convex fuzzy set, strongly convex fuzzy set, strictly convex fuzzy set and closed fuzzy set.

Theorem

A fuzzy set B on R is said to be convex if and only if

$B(\lambda x_1 + (1-\lambda)x_2) \geq min(B(x_1), B(x_2))$.for all x_1 *and* x_2 in R and $\lambda \in (0, 1]$.

Proof: Suppose that fuzzy set B is convex.

To prove that $B(\lambda x_1 + (1-\lambda)x_2) \geq min(B(x_1), B(x_2))$ for all x_1 *and* x_2 in R and $\lambda \in (0, 1]$.

Suppose that B is not a convex fuzzy set.

Then there exist some β such that B_β is not convex.

Let $x_1, x_2 \in B_\beta$ but $\lambda x_1 + (1-\lambda)\, x_2 \notin B_\beta$ ..... (1)

Choose $\beta = B(X_1) \leq B(x_2)$, for some x_1 and x_2 in R.

Given that $B(\lambda x_1 + (1-\lambda)x_2) \geq min(B(x_1), B(x_2))$.for all x_1 and x_2 in R and

$\lambda \in (0, 1]$.

That is $B(\lambda x_1 + (1-\lambda)x_2) \geq \beta = B(x_1) = min(B(x_1), B(x_2))$.

$\rightarrow \lambda x_1 + (1-\lambda)x_2 \in B_\beta$.

A Contradiction to equation (1).

Therefore, B_β is convex set. $\forall$ $\beta \in (0, 1]$.

Conversely, suppose that B is Convex Fuzzy set on R.

To prove that, $B(\lambda x_1 + (1-\lambda)x_2) \geq min(B(x_1), B(x_2))$ for all x_1 and x_2 in R and $\lambda \in$

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(0, 1].

Suppose that for some y_1 and $y_2 \in B_\beta \subset R$ and $\Lambda \in (0, 1]$.

Implies that $B(y_1) \geq \beta$ and $B(y_2) \geq \beta$, since for every $\beta \in (0, 1]$, B_β is convex.

We have, $B(\Lambda y_1 + (1 - \Lambda)y_2) \in B_\beta$. A contradiction to Assumption.

Therefore, $B(\lambda x_1 + (1 - \lambda)x_2) \geq min(B(x_1), B(x_2))$.for all x_1 and x_2 in R and $\lambda \in (0, 1]$.

Theorem

If B_1 and B_2 are convex fuzzy sets, then Intersection of B_1 and B_2 is convex.

Note that Converse of the above theorem is not true. Union of two convex fuzzy sets need not be convex.

Theorem

Let B be a convex fuzzy set on R. If there exist $b \in J$, for every pair of distinct points $x \in B$ and $y \in B$ implies that $B(bx_1 + (1 - b)x_2) > min(B(x_1), B(x_2))$.

Then B is strongly convex fuzzy set on R.

Proof: Suppose that B is not a strongly convex fuzzy set on R.

Then there exists $x_1, y_1 \in R$, $x_1 \neq y_1$, $c \in J$ such that

$B(cx_1 + (1 - c)x_2) \leq min(B(x_1), B(x_2))$..... (1)

Let $z_1 = cx_1 + (1 - c)x_2$), Then from (1) we have,

$B(z_1) \leq min(B(x_1), B(x_2))$. ..... (2)

Since B is convex fuzzy set on R.

$B(z_1) \geq min(B(x_1), B(x_2))$. ..... (3)

from (2) and (3) implies that,

$B(z_1) = min(B(x_1), B(x_2))$. ..... (4)

Choose γ_1 and γ_2 such that $0 < \gamma_1 < c < \gamma_2 < 1$. Then c is point on the line joining γ_1

and γ_2. Therefore we can write $c = b\gamma_1 + (1 - b)\gamma_2$ for some b $\in (0,1]$.

Let $x' = \gamma_1 x_1 + (1 - \gamma_1)y_1$ and $y' = \gamma_2 x_1 + (1 - \gamma_2)y_1$ ..... (5)

Thus, $bx' + (1 - b)y' = b(\gamma_1 x_1 + (1 - \gamma_1)y_1) + (1 - b)(\gamma_2 x_1 + (1 - \gamma_2)y_1)$.

$$= (b\,\gamma_1 x_1 + (1 - b)\,\gamma_2 x_1) + [b(1 - \gamma_1)y_1 + (1 - b)(1 - \gamma_2)y_1]$$

$$= (b\,\gamma_1 + (1 - b)\,\gamma_2)x_1 + [b - b\,\gamma_1 + 1 - \gamma_2 - b + b\,\gamma_2]y_1$$

$$= cx_1 + [b\,\gamma_1 - \gamma_2 + b\,\gamma_2 + 1]y_1 = cx_1 + (1 - c)y_1 = z_1.$$

Therefore, $bx' + (1 - b)y' = cx_1 + (1 - c)y_1 = z_1$.

From (5), we have, $B(x_1) \geq min(B(x_1), B(y_1))$……..(6)

$B(y_1) \geq min(B(x_1), B(y_1))$..... (7)

Since B is convex fuzzy set on R.

According to the hypothesis and from (6) and (7) and $c = b\gamma_1 + (1 - b)\gamma_2$.

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$B(z_1) > min(B(x_1), B(y_1))$

$\geq min((B(x_1), B(y_1)), (B(x_1), B(y_1)) = min(B(x_1), B(y_1))$.

Which is contradiction to (4).

Contradiction arises due to our assumption that B is not a strongly convex fuzzy set.

Hence B is strongly convex fuzzy set for all $c \in J$.

Theorem

Let B be a convex fuzzy set on R. If there exists $b \in J$, for every $x_1, y_1 \in R$, $B(x_1) \neq B(y_1)$ implies that $B(bx_1 + (1 - b)y_1) > min(B(x_1), B(y_1))$……...(*)

Then B is strictly convex fuzzy set on R.

Proof: Suppose that B is not a strictly convex fuzzy set on R.

Then there exists $b \in J$ and $x_1, y_1 \in R$ and $B(x_1) \neq B(y_1)$ implies that,

$B(bx_1 + (1 - b)y_1) \leq min(B(x_1), B(y_1))$..... (1)

Without loss of generality we suppose that, $B(x_1) > B(y_1)$.

Let $z_1 = bx_1 + (1 - b)y_1$. Thus, $b(z_1) \leq min(B(x_1), B(y_1))$.

Since, B is convex fuzzy set, by definition of convexity, we have,

$B(z_1) \geq min(B(x_1), B(y_1))$. ..... (2)

(1) and (2) implies that,

$B(z_1) = min(B(x_1), as B(y_1)) < B(x_1)$..... (3)

$B(bx_1 + (1 - b)z_1) > Min(B(x_1), B(z_1)) = B(z_1)$,

Consider. $B(b^2x_1 + (1 - b^2)z_1)$.

$=B(b^2x_1 + (1 - b)z_1(1 + b)) = B(b^2x_1) + b(1 - b)z_1 + (1 - b)z_1)$

$=B(b(bx_1 + (1 - b)z_1) + (1 - b)z_1)$.

$> min(B(bx_1 + (1 - b)z_1), B(z_1)) > min(B(x_1), B(z_1), B(z_1)) = B(z_1)$.

Now, Assume that above statement is $B(b^2x_1 + (1 - b^2)z_1) > B(z_1)$ is true for some k.

That is $B(b^kx_1 + (1 - b^k)z_1) > B(z_1)$.

We have to show that above statement is true for k+1.that is to prove that,

$B(b^{k+1}x_1 + (1 - b^{k+1})z_1) > B(z_1)$.

Consider, $B(b(b^kx_1 + (1 - b^k)z_1) + (1 - b)z_1) = B(b^{k+1}x_1 + b(1 - b^k)z_1 + (1 - b)z_1)$.

$=B(b^{k+1}x_1 + z_1(b(1 - b^k) + (1 - b)z_1)) = B(b^{k+1}x_1 + z_1(b - b^{k+1} + 1 - b))$.

$=B(b^{k+1}x_1 + z_1(1 - b^{k+1})) = B(b^{k+1}x_1 + (1 - b^{k+1})z_1)$.

$> min(B(b^kx_1 + (1 - b^k)z_1), B(z_1))$.

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$> min(B(z_1), B(z_1)) = B(z_1)$ ..... (By Assumption)

Therefore, by first principle of Mathematical induction, we have,

$B(b^k x_1 + (1 - b^k)z_1) > B(z_1)$.for all $k \in N$ ..... (4)

Now let $z_1 = cx_1 + (1 - c)\, y_1$.

Put above value of z_1 in $b^k x_1 + (1 - b^k)\, z_1$.

Therefore, $b^k x_1 + (1 - b^k)(cx_1 + (1 - c)y_1) = b^k x_1 + cx_1 + (1 - c)y_1 - b^k(cx_1 + (1 - c)y_1)$.

$= b^k x_1 + cx_1 + (1 - c)y_1 - b^k cx_1 - b^k y_1 + b^k cy_1 = (b^k + c - b^k c)x_1 + (1 - c - b^k + b^k c)y_1$.

$= (b^k + c - b^k c)x_1 + (1 - (c + b^k - b^k c))$.

put $b^k + c - b^k c = e$. Therefore, $b^k x_1 + (1 - b^k)z_1) = ex_1 + (1 - e)y_1$.....(5)

Let $k_1 \in N$ such that, $b^{k1} + \frac{1}{1-b} < \frac{c}{1-c}$. Therefore, $(1 - c)(b^{k_1} + \frac{1}{1-b}) <$ c.

$= (1 - c)b^{k_1} + (1 - c)\frac{1}{1-b} < c. = b^{k_1} - b^{k_1} c + \frac{1-c}{1-b} <$ c.

Thus, $0 < c - b^{k_1} + b^{k_1} c + \frac{1-c}{1-b}$.

i.e.$0 < c - b^{k_1}$(1-c)+ $\frac{1-c}{1-b}$ .Now take, $c - b^{k_1}$(1-c)+ $\frac{1-c}{1-b}$ =f.

Let $x_1 = ex + (1 - e)\, y$ and $y_1 = fx + (1 - f)\, y$.

Therefore, $b^{k_1} x + (1 - b^{k_1})\, z_1 = ex + (1 - e)\, y = x_1$ ..... (6)

According to (4) and (6), we obtain,

$B(x_1) = B(ex + (1 - e)y)$

$= B(b^{k_1} x + (1 - b^{k_1})\, z_1) > B(z)$.....(7)

Now two cases arises on $B(x_1)$ and $B(y_1)$

Case(i) If $B(x_1) \leq B(y_1)$.

Since, $z_1 = cx + (1 - c)\, y = bx_1 + (1 - b)\, y_1$.

Since B is convex fuzzy set on R implies that,

$B(z_1) \geq Min(B(x_1), B(y_1)) = B(x_1)$..... (Since, $B(x_1) \leq B(y_1)$).

A contradiction to $B(x_1) > B(z_1)$. Therefore, $B(x_1) \leq B(y_1)$ is not true.

Case(ii) If $B(x_1) > B(y_1)$.

$z = cx + (1 - c)\, y = bx_1 + (1 - b)\, y_1$.

and $B(x_1) \neq B(y_1)$, thus hypothesis implies that,

$B(z_1) > min(B(x_1), B(y_1)) = B(y_1)$. ..... (8)

Also, since $x_1 = ex + (1 - e)\, y$ and $y_1 = fx + (1 - f)y$ and B is convex fuzzy

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set,

We will have, $B(x_1) \geq min(B(x), B(y))$ and $B(y_1) \geq min(B(x), B(y))$.

From equation (8) and above two equations, we have,

$B(z_1) > min(B(x_1), B(y_1))$.

$\geq min(min((B(x), B(y)), min(B(x), B(y))) = min(B(x), B(y))$.

Which is a contradiction to hypothesis.

Therefore, our assumption that there exist $c \in J, x, y \in R$ such that

$B(z_1) \leq min(B(x), B(y))$ is wrong. B is strictly convex fuzzy set on R.

Corollary 2.

Let B be a fuzzy closed set on R. If there exists $b \in J$, for every $x_1, y_1 \in R$, $B(x_1) \neq B(y_1)$ implies that $B(bx_1 + (1 - b)y_1) > min(B(x_1), B(y_1))$……...(*)

Then B is strictly convex fuzzy set on R.

Theorem

Let B be a strictly convex fuzzy set on R. If there exist $b \in J$ such that for every pair of distinct points $x_1, y_1 \in R$, we have, $B(bx_1+(1-b)y_1) > min(B(x_1), B(y_1))$. Then B is a strongly convex fuzzy set on R.

Proof : Given that B is strictly convex fuzzy set on R.

That is for every distinct pair of $x_1, y_1 \in R$, $x_1 \neq y_1$

We have,

$B(cx_1 + (1 - c)y_1) > min(B(x_1), B(y_1))$, for $B(x_1) \neq B(y_1)$ and $\forall c \in J$.

So, it is enough to prove that, for $x_1 \neq y_1, x_1, y_1 \in R$ and $B(x_1) = B(y_1)$,

$B(cx_1 + (1 - c)y_1) > min(B(x_1), B(y_1))$, $\forall c \in J$.

From the hypothesis, $B(x_1) = B(y_1)$ implies that,

$B(bx_1 + (1 - b)y_1) > min(B(x_1), B(y_1)) = B(x_1) = B(y_1)$.

For each $x_1, y_1 \in R, x_1 \neq y_1$. let $x' = bx_1 + (1\text{-}b)y_1$.

Implies that, $y_1 = \frac{x'}{1-b} - \frac{bx_1}{1-b}$.

For each $c \in J$.

case(i) $b < c$. we have,

$$cx_1 + (1- c)y_1 = cx_1 + (1- c)\left(\frac{x'}{1-b} - \frac{bx_1}{1-b}\right).$$

$$= cx_1 + \frac{x'}{1-b} - \frac{cx'}{1-b} - \frac{bx_1}{1-b} + \frac{cbx_1}{1-b} = [c - \frac{b}{1-b} + \frac{cb}{1-b}]\, x_1 + \frac{1-c}{1-b} x'.$$

$$= \left(\frac{c(1-b)-b+bc}{1-b}\right)x_1 + \left(\frac{1-c}{1-b}\right)x' = \left(\frac{c-bc-b+bc}{1-b}\right)x_1 + \left(\frac{1-c}{1-b}\right)x'.$$

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$= (\frac{c-b}{1-b})x_1+(1-\frac{c-b}{1-b})x' = dx_1+(1-d)x'$, where, $d=\frac{c-b}{1-b}$.

as $0 < c < 1$ and $0 < b < 1$ implies that $0< c-b < 1-b <1$.

$0 < \frac{c-b}{1-b} <1$.Therefore, $B(z_1) > \min(B(x_1),B(x'))$.

By hypothesis that B is strictly convex fuzzy set on R.

Case(ii) c < b.

$Cx_1+(1-c)y_1=c\,(\frac{x'}{b}-\frac{(1-b)y_1}{b})+(1-c)y_1= c\ \frac{x'}{b}-\frac{c(1-b)y_1}{b}+(1-c)y_1$.

$= c\ \frac{x'}{b}-[\frac{c(1-b)_1}{b}+(1-c)\ ]y_1 = c\ \frac{x'}{b}-[\frac{c-bc-b+bc}{b}]\ y_1$

$= c\ \frac{x'}{b}-\frac{c-b}{b}y_1 .= c\ \frac{x'}{b}-(-1+\frac{c}{b})y_1 = c\ \frac{x'}{b}-(\frac{c}{b}-1)y_1$. Since B is strictly convex fuzzy set on R. ∴ In any case, $B(cx_1 + (1 - c)y_1) = B(dx' + (1 - d)y_1)$

$> min(B(x'), B(y_1)) = B(x_1) = B(y_1)$,

Where $x \neq y$, and $d=\frac{c}{b}$ Therefore, B is a Strongly convex fuzzy set on R.

Theorem

A fuzzy set B on R is closed if and only if fuzzy hypograph,

f.hypo(B) $=\{ (y, t') : y \in R, t' \in (0, B(x))\}$ is a closed subset of $R \times (0, 1]$.

Proof: Suppose that B is a closed fuzzy set on R.

Hence B is upper semi continuous on R.

let (x_n', y_n') be a sequence of points of f.hypo(B) that converges to (x',y') in Rx(0, 1].

Since, B is upper semi continuous and since $B(x_n') \geq y_n'$,

We must have, $B(x') = \lim_{n\to\infty} B(x_n') \geq \lim_{n\to\infty} y_n' = y'$.

By definition of fuzzy hypograph $(x', y') \in f.hypo(B)$. Therefore, f.hypo(B) is a closed subset of $R \times (0, 1]$.

Conversely, suppose that f.hypo(B) is a closed subset of $R\times (0, 1]$. For $\alpha' \in (0, 1]$, two cases arise. First one is $\alpha'-cut$ of B is empty or finite then clearly it is closed subset of $R \times (0, 1]$.

Second case is $\alpha'-cut$ of B has infinitely much elements.

to show that it is closed. Consider any sequence y_n in $\alpha' - cut$ of B that converges to y'.

Then $B(x_n) \geq \alpha'$. The sequence of elements (y_n, α') of f.hypo(B) which is converges to (y', α'). Since, f.hypo(B) is closed.$(y', \alpha') \in$ f.$hypo(B)$ that is $B(y') \geq \alpha'$. It implies that y' is in the $\alpha' - cut$ of B. Therefore, $\alpha' - cut$ of B is closed. It follows that B is

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closed.

Theorem

Let B be a convex fuzzy set defined on R. Then B is convex if and only if for all x_1, $x_2 \in supp(B)$, there exists a $\beta \in (0, 1)$,(β depends on $x_1$and x_2) such that

$B(\beta x_1 + (1 - \beta)x_2) \geq \beta B(x_1) + (1 - \beta)B(x_2)$.

Proof: To show that B is convex it is enough to prove that f.hypo(B) is convex subset of

$R \times (0, 1]$.

Let if possible suppose that f.hypo(B) is not convex then there exists

$(x_1, s_1), (x_2, s_2) \in f.hypo(B)$ and $\beta \in (0, 1)$ such that $q' = \beta(x_1, s_1) + (1 - \beta)(x_2, s_2) \notin f.hypo(B)$. Now let $T_1 = f.hypo(B) \cap J_1$ and $T_2 = f.hypo(B) \cap J_2$.

Where, $J_1 = \beta_1(x_1, s_1) + (1 - \beta_1)t_0 : \beta_1 \in [0, 1]$ and $J_2 = \beta_1 t_0 + (1 - \beta_1)(x_2, s_2) : \beta_1 \in [0, 1]$.

Since, B is closed fuzzy set. Therefore, T_1 and T_2 are closed and bounded subset of $R \times (0, 1]$.

$t_0 \notin T_1$ and $t_0 \notin T_2$ thus there exists $t_1 = (x', u) \in T_1$ and $t_2 = (y', v) \in T_2$. Where

$x', y' \in supp(B)$ and $u, v \in (0, 1]$ such that $\min_{b_1} \|b_1 - t_0\| = \|z_1 - t_0\|$.

$\min_{b_2} \|b_2 - t_0\| = \|z_2 - t_0\|$. Where b_1 is in T_1 and b_2 is in T_2.

Hence, we have, $f.hypo(B) \cap (z_1, t_0] = \emptyset$ and $f.hypo(B) \cap [t_0, z_2) = \emptyset$.

Where, $(z_1, t_0] = \beta_1 z_1 + (1 - \beta_1)t_0 : \beta_1 \in (0, 1]$ and $[t_0, z_2) = \beta_1 t_0 + (1 - \beta_1)z_2 : \beta_1 \in (0, 1]$.

Therefore, $f.hypo(B) \cap (z_1, z_2) = \emptyset$

Also, by given, there exist $\beta \in (0, 1)$ such that

$B(\beta x_1 + (1 - \beta)x_2) \geq \beta B(x_1) + (1 - \beta)B(x_2)$..... (1)

Since, $(x', u), (y', v) \in f.hypo(B)$.we have,

$B(x') \geq u$ and $B(y') \geq v$, ... (2)

From (1) and (2), we obtain,

$B(\beta x' + (1 - \beta)y') \geq \beta B(x') + (1 - \beta)B(y'))$.

That is $\beta z_1 + (1 - \beta)z_2 = \beta(x', u) + (1 - \beta)(y', v) \in f.hypo(B)$.

This contradicts that $f.hypo(B) \cap (z_1, z_2) = \emptyset$.

Our assumption was wrong.

Therefore, f.hypo(B) is convex fuzzy set. Hence B is convex fuzzy set.

Theorem

Let B be a closed fuzzy set defined on R and there exist $\beta \in [0, 1]$ such that for all x_1, $y_1 \in R$, $s, t \in (0, 1)$ such that $B(x_1) > s$ and $B(y_1) > t$, $B(\beta x_1 + (1 - \beta)y_1) > \beta s + (1 - \beta)t$.

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Then B is convex fuzzy set on R.

Proof: To show that B is convex fuzzy set on R.

It is enough to show that $\forall x_1, y_1 \in R$ there exist $\beta \in (0, 1)$ such that

$B(\beta x_1 + (1-\beta)y_1) \geq \beta B(x_1) + (1-\beta)y_1$.

Suppose on the contrary that there exist, $x_1, y_1 \in R$ such that $\forall \beta \in (0, 1)$,

$B(\beta x_1 + (1-\beta)y_1) < \beta B(x_1) + (1-\beta)y_1)$. ..... (1)

for $\epsilon' > 0$. since $B(x_1) > B(x_1) - \epsilon'$ and $B(y_1) > B(y_1) - \epsilon'$. By the hypothesis,

We have, $B(\beta x_1 + (1-\beta)y_1) \geq \beta B(x_1 - \epsilon') + (1-\beta)(y_1 - \epsilon')$

$=\beta B(x_1) + (1-\beta)(y_1) - \epsilon'$.

Since, ϵ' is arbitrary. It follows that, $B(\beta x_1 + (1-\beta)y_1) \geq \beta B(x_1) + (1-\beta)(y_1)$.

Which is contradiction to (1). Therefore, B is convex fuzzy set on R.

Theorem

Let B be a convex fuzzy set defined on R. If there exists $\beta \in (0, 1)$ such that for every non equal elements $y_1, y_2 \in supp(B)$. $B(\beta y_1 + (1-\beta)y_2) > \beta B(y_1) + (1-\beta)B(y_2)$……(*) holds. Then B is a strictly convex fuzzy set on R.

Proof: Suppose that B is not a strictly convex fuzzy set.

Then there exists $y_1, y_2 \in supp(B), y_1 \neq y_2$ and $\beta' \in (0, 1)$ such that, let $z_1 = \beta' y_1 + (1-\beta') y_2$

$B(z_1) = \beta' B(y_1) + (1-\beta') B(y_2)$. ..... (1)

By the hypothesis, It is clear that, $\beta \neq \beta'$.

Case(i) $0<\beta'<\beta$.

Let $z_2 = \frac{\beta'}{\beta} y_1 + (1-\frac{\beta'}{\beta}) y_2$ ……..(2)

Then, $z_1 = \beta' y_1 + (1-\beta') y_2 = \beta(\frac{\beta'}{\beta} y_1 + (1-\frac{\beta'}{\beta}) y_2) + (1-\beta) y_2 = \beta z_2 + (1-\beta) y_2$.

According to equation (*), we have, $B(z_1) > \beta B(z_2) + (1-\beta) B(y_2)$.

Since B is convex fuzzy set defined on R.

Equation (2) and above inequality implies that,

$B(z_1) > \beta' B(y_1) + (1-\beta') B(y_2)$.

Which is contradiction to equation (1).

Case(ii) If $\beta<\beta'<1$.that is $0< \frac{\beta'-\beta}{1-\beta} <1$. Let $z_3 = \frac{\beta'-\beta}{1-\beta} y_1 + \frac{1-\beta'}{1-\beta} y_2$……(3)

Thus, $z_1 = \beta' y_1 + (1-\beta') y_2 = \beta y_1 + (1-\beta) z_3$.

Since B is convex fuzzy set on R. equation (3) and above inequality becomes,

$B(z_1) > \beta' B(y_1) + (1-\beta') B(y_2)$. Contradiction to equation (2).

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Therefore, B is strictly convex fuzzy set on R.

Conclusion

The convexity of fuzzy sets and its many types and features are the main topics of this survey study. Further research in the area of convex fuzzy sets is beneficial. The concept of convexity will be applied more broadly to fuzzy relations in future research, and different types of fuzzy relations, such as Normal fuzzy relations and Closed fuzzy relations with its convexity, will be studied separately. Generalizing the definition of fuzzy hypograph of fuzzy set to fuzzy relation would be helpful to study closed fuzzy relations. Numerous applications of the study of convexity in generalizing ways can be found in the fields of science, business, and the arts to optimize the resolution of any problem.

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